**A Unified Ontology of Physics: Mathematical Construction of the ABC Field Theory and Emergence of Fundamental Equations**

**Authors**: Li Zhijun, Zhao Guangyao

**Abstract:**

This paper aims to rigorously construct the Li Zhijun ABC Cosmic Vortex Field Theory (hereafter ABC Field Theory) mathematically and demonstrate its necessity as the common ontological foundation for the core equations of modern physics. The central thesis is that the phenomena described by quantum mechanics (Schrödinger equation, Dirac equation), quantum field theory (Yang-Mills equations), quantum statistics (von Neumann equation), and even quantum gravity (Wheeler-DeWitt equation) are all dynamic emergences from the combinations of the A-field (electromagnetic vortex), B-field (color charge vortex), and C-field (Higgs vortex) under different approximate conditions. The breakthrough progress of this paper lies in:

1. Rigorous Mathematical Definition: For the first time, providing a strict mathematical formulation of the ABC fields: The A-field is a connection on a U(1) principal bundle, its topological charge being the Chern number; the B-field is a connection on an SU(3) principal bundle, its topological charge being the winding number around ; the C-field is a connection on an SU(2) principal bundle, its topological charge being the degree of the map to . Particle states are described by their tensor product states, with quantum numbers determined by the corresponding topological charges.
2. First-Principles Derivation: Starting from the dynamic principles of the combined ABC field states, and by introducing the generalized covariant derivative , naturally deriving the Schrödinger equation, Dirac equation, Yang-Mills equations, and Klein-Gordon equation as approximate forms of its equations of motion under different limits.
3. Proof of Symmetry Origin: Demonstrating that the , , and gauge symmetries originate from the topological invariance requirements of the base manifolds of the A, B, and C fields, respectively, thereby geometrically explaining the origin of symmetries.
4. Demonstration of Unification: Incorporating complex system issues such as quantum chaos and many-body localization into the framework, treating them as dynamic problems of ABC field combination states under specific potentials, showcasing the theory’s unparalleled unifying power.

This work develops the ABC theory from a qualitative framework into a mathematically rigorous physical theory, providing a solid ontological foundation and a clear mathematical path for the ultimate construction of a theory of everything.

**Keywords:** ABC Field Theory; Fiber Bundle; Topological Charge; Chern Number; Winding Number; Gauge Symmetry; Covariant Derivative; Unified Theory

1. **Introduction: Towards a Mathematically Rigorous Unified Ontology**

Modern physics is built upon a series of outstanding yet seemingly disparate equations. A fundamental question remains unanswered: Are these equations fundamental? Or are they emergent approximations of a deeper, more unified reality?

The Li Zhijun ABC theory provides a blueprint for answering this question. Previous works outlined the physical picture but lacked mathematical rigor. This paper aims to bridge this critical gap. We will endow the ABC fields with precise mathematical definitions and derive the core equations of physics from first principles.

We will demonstrate that the ABC Field Theory does not compete with existing theories but provides them with an ontological foundation and mathematical root. The Schrödinger wave function, the Dirac spinor, and the Yang-Mills gauge field are all specific representations of the combined states of the ABC fields. The grand unification of physics is not about discovering a new equation, but realizing that all equations originate from the same geometric reality.

1. **Rigorous Mathematical Definition of the ABC Fields and Topological Charges**

We abandon the vague metaphor of “vortex” and adopt the precise language of differential geometry to define the ABC fields.

2.1 A-field (Electromagnetic Vortex Field): U(1) Principal Bundle Connection and Chern Number

* Mathematical Definition: The A-field is a connection on a U(1) principal fiber bundle . Here, the base manifold M is spacetime, and the fiber is the U(1) group.
* Topological Charge (Chern Number): The global topology of the A-field is characterized by the first Chern class:

where is the curvature of the connection . This integer is the mathematical root of charge quantization.  
\* Dynamic Variable: Locally, the connection is represented as a 1-form , which is the dynamic variable.

2.2 B-field (Color Charge Vortex Field): SU(3) Principal Bundle Connection and Winding Number

* Mathematical Definition: The B-field is a connection on an SU(3) principal fiber bundle .
* Topological Charge ( Winding Number): The center of the SU(3) group is isomorphic to (the cyclic group of order three). The topological stability of the B-field is described by its winding number around , which takes values in . This determines the triple nature of color charge and color confinement (all observables must be in the trivial representation of ).
* Dynamic Variable: The connection , where , and are the generators of SU(3).

2.3 C-field (Higgs Vortex Field): SU(2) Principal Bundle Connection and Degree

* Mathematical Definition: The C-field is a connection on an SU(2) principal bundle . Since the SU(2) group is homeomorphic to the 3-sphere , the fiber of its associated bundle can be viewed as .
* Topological Charge (Degree): The topology of the C-field is characterized by the degree of the map : M , which is related to chirality and mass generation.
* Dynamic Variable: The connection .

2.4 Particle States: Joint Spectrum and Combined States of ABC Connections

An elementary particle is no longer a point-like object, but a common eigenstate of the ABC connections, labeled by their topological quantum numbers:

For example:  
\* Electron: ( is the trivial representation of )

* Up Quark: ( is the fundamental representation of )
* Photon: (topologically trivial excitation)

1. **Dynamic Principles and Emergence of Fundamental Equations**

Physical laws arise from the dynamics of the ABC connections.

3.1 Generalized Covariant Derivative and Equations of Motion

The dynamics of the entire system is governed by a generalized covariant derivative:

It is the covariant differential for all internal degrees of freedom.

The motion of the system is determined by a Yang-Mills-Higgs type action:

where are the curvatures of the ABC connections, respectively, and is the interaction term. Varying this action yields the generalized Yang-Mills equations:

This is a set of coupled, nonlinear partial differential equations describing the complete dynamics of the ABC fields.

3.2 Emergence of Core Equations (Derivation Examples)

Example 1: Emergence of the Schrödinger Equation  
Consider the low-energy, non-relativistic limit, with only the A-field significant. The generalized covariant derivative reduces to:

Acting on a scalar wavefunction dominated by the A-field (representing the state), its equation of motion approximates to:

This is precisely the covariant form of the Schrödinger equation. Here, the wavefunction is the amplitude of a specific mode of the A-field connection.

Example 2: Emergence of the Dirac Equation  
In the relativistic case, consider the A-field and C-field. The Dirac spinor is not a fundamental field, but the wavefunction of the joint eigenstate of the coupled modes of the A and C fields. Its equation of motion naturally separates from the generalized YM equations:

where (the background value of contributes to the mass m). This is the Dirac equation.

Example 3: Emergence of the Yang-Mills Equations  
In the pure B-field sector, the generalized equation of motion directly reduces to:

This is exactly the Yang-Mills equation. The current comes from the coupling of other fields to the B-field.

Example 4: Emergence of the Klein-Gordon Equation  
In the scalar field sector, the equation reduces to:

namely, the Klein-Gordon equation.

1. **Geometric Origin of Symmetries**

Gauge symmetry is no longer a fundamental assumption, but a necessary consequence of the geometry of the ABC fields.  
\* Symmetry: Originates from the gauge invariance of the A-field’s base manifold (U(1) bundle). The transformation leaves the curvature invariant, thus preserving the dynamics.

* Symmetry: Originates from the gauge invariance of the B-field’s base manifold (SU(3) bundle). Its center directly corresponds to the topological charge of the B-field.
* Symmetry: Originates from the gauge invariance of the C-field’s base manifold (SU(2) bundle).

These symmetries are local gauge freedoms under global topological constraints, rooted in the geometric structure of the fiber bundles inhabited by the ABC fields.

1. **ABC Field Theory Perspective on Complex Systems**

5.1 Quantum Chaos and Many-Body Localization  
A many-body system is a collection of numerous combined states of the ABC fields. Its Hamiltonian is:

where is the free Hamiltonian of the i-th combined state, and is the interaction between them, described by the cross terms in the generalized covariant derivative.  
\* Quantum Chaos: When the interaction is strong, intense mixing and entanglement occur between different ABC combined states, leading to rapid thermalization of the system, with energy level statistics satisfying the Wigner-Dyson distribution.

* Many-Body Localization: When strong disorder exists (e.g., random distribution of ), even with interactions, the ABC combined states become localized near their specific topological modes, unable to effectively couple with distant states, preventing thermalization and maintaining quantum coherence.

5.2 Quantum Statistics and the von Neumann Equation  
The system’s density matrix describes the statistical distribution of the ensemble of ABC field combined states. Its evolution is described by the quantum Liouville equation:

In an open system, interaction with the environment leads to decoherence, and the equation becomes the von Neumann equation in Lindblad form:

where the Lindblad operators represent quantum jump operations induced by the environment on the system’s ABC field combined states.

1. **Conclusion and Outlook: Towards Quantum Gravity**

This paper has, for the first time, constructed the ABC Field Theory as a mathematically rigorous physical theory. By strictly defining the A, B, and C fields as connections on different fiber bundles, we have successfully emergent the core equations of quantum mechanics and quantum field theory from their dynamics. Gauge symmetries are shown to originate from the geometric invariance of the fiber bundles.

**Outlook:**  
The most fascinating next step is to apply this framework to gravity. The gravitational field can be geometrically defined as the metric of the spacetime base manifold M. A natural hypothesis is that the metric is determined by some kind of condensation or combination of the ABC connections:

Then, the Einstein field equations might also emerge from the generalized equations of motion of the ABC fields. The Wheeler-DeWitt equation could potentially serve as the constraint equation describing the quantum state of the entire universe’s ABC fields.

This provides a clear and concrete path towards unifying gravity with quantum mechanics: within the framework of ABC theory, gravity and the other three forces stand, for the first time, on the same mathematical level—they are all dynamic manifestations of the ABC connections. The distinction lies only in that gravity is the dynamics of spacetime geometry, while the other forces are the dynamics of the geometry of internal fiber bundles. A comprehensive theory of everything might take the form of the generalized Yang-Mills equation given in Sec. 3.1.

This research transforms the ABC theory from a revolutionary idea into a feasible physical theory framework with a rigorous mathematical foundation and powerful predictive and deductive capabilities.

**References**

[1] Nash, C. & Sen, S. Topology and Geometry for Physicists. Academic Press (1983).  
[2] Nakahara, M. Geometry, Topology and Physics. Taylor & Francis (2003).  
[3] Peskin, M. E. & Schroeder, D. V. An Introduction to Quantum Field Theory. Westview Press (1995).  
[4] Original references for equations listed in Figures 1 and 2.  
[5] Li, Z. J. “On the Fundamental Vortex Fields of the Universe.” Preprint (2023).